Work, Energy and Power

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Note : The notes given in this file is no substitute to the much detailed discussion held in the online/contact classes with active participation of students. It , at best, serves the purpose of ready reference for important concepts/derivations covered in the classes.

Energy

Energy is defined as the capacity or ability of a body or a system to do work.
It is a scalar quantity

- Kinetic energy
- Potential energy
- Heat energy
- Light energy
- Sound energy
- Chemical energy

Kinetic energy

It is the energy possessed by a body by the virtue of its motion.

$$KE = \frac{1}{2}mv^2$$

- Kinetic energy is a scalar quantity (does not depend on direction of motion)
- SI unit of kinetic energy is joule (J)
- Kinetic energy can be either zero (for an object at rest) or positive (for an object in motion). *KE* can never be negative.

Examples

- Cricket ball bowled by a bowler
- A truck moving on a highway
- An electron moving around the nucleus
- Electrons moving in a conductor carrying current
- Communication satellite moving around the earth

Kinetic energy (in terms of linear momentum)

Kinetic energy is given by

$$KE = \frac{1}{2}mv^2$$

Multiplying and dividing by mass of the body (m) we get

$$KE = \frac{1}{2}mv^2 \frac{m}{m}$$

$$KE = \frac{1}{2}\frac{m^2v^2}{m}$$

Linear momentum is given by $p = mv$ therefore we get

$$KE = \frac{p^2}{2m}$$

Potential energy

It is the energy possessed by a system by the virtue of its position or state or configuration.

- Potential energy is a scalar quantity
- SI unit of potential energy is joule (J)
- Potential energy may be positive, negative or zero

Examples

- A spring that is compressed (or elongated) has elastic potential energy
- Two or more atoms forming a molecule have electrostatic potential energy Umpire standing on the ground and the winners trophy placed on the table
- Water stored in a dam
- Communication satellite in its parking orbit around the earth

There is no FIXED formula for potential energy !

Work done by certain forces is stored in the system in the form of PE and is converted back into kinetic energy once the constraints are removed

Conservative forces

- Work done by the force in a closed path is zero
- Work done by the force is independent of the path followed and depends only on initial and final positions
- Force is derivable as a negative gradient of scalar potential energy

$$\boldsymbol{F} = -\text{grad}\, U$$

$$\boldsymbol{F} = -\overline{\nabla}\, U$$

$$\boldsymbol{F} = -\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) U$$

- Examples : gravitational force, spring force, electrostatic force

Non-conservative forces

- Work done by the force in a closed path is not zero
- Work done by the force is depends on the path followed by the body
- Force is not derivable as a negative gradient of scalar potential energy
- Examples : frictional force, viscous force

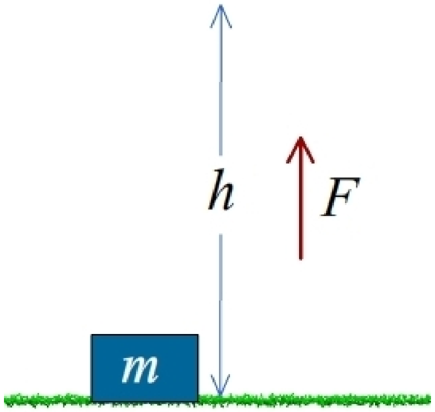
Potential energy of a body raised from ground

Consider a body of mass m on the ground. Let the body be lifted, very slowly, to a height h from the ground. Force required to lift the body (i.e. to overcome the gravitational force on the body) is mg (upwards) therefore work done by the applied force in lifting the body up is

$$W = mg \times h \times \cos(0)$$

$$W = mgh \times 1$$

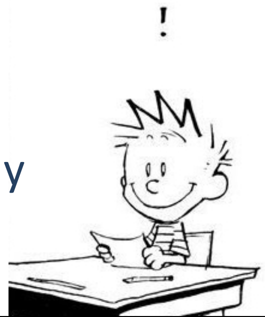
$$W = mgh$$



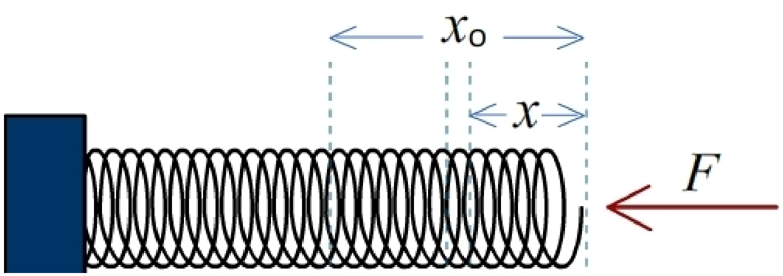
Work done by the external agent on the body is stored in the form of potential energy in it.

$$\boxed{PE_G = mgh}$$

The fact that work done is stored as energy can be verified by leaving the body from that height. The body gains kinetic energy as it reaches the ground !



Potential energy stored in a spring



Consider a spring of spring constant k. From the initial relaxed state the spring is compressed through a distance x_o. Work in causing a small compression (dx) is

$$\mathrm{d}W = F \times \mathrm{d}x \times \cos(0)$$

$$\mathrm{d}W = F\ \mathrm{d}x$$

Force applied externally (to overcome the restoring force by the spring) is kx.

$$\mathrm{d}W = kx\ \mathrm{d}x$$

Total work done by external force in causing a compression of x_o is

$$\int \mathrm{d}W = \int_0^{x_o} kx\ \mathrm{d}x$$

$$W = k\int_0^{x_o} x^1\ \mathrm{d}x$$

$$W = k\left[\frac{x^2}{2}\right]_0^{x_o}$$

$$W = k\left[\frac{x_o^{\ 2}}{2} - \frac{0}{2}\right]$$

This work done is stored as (elastic) *PE* in the spring

$$\boxed{PE = \frac{1}{2}kx_o^{\ 2}}$$

Work energy theorem

Work done by all the forces acting on a body is equal to the change in its kinetic energy.

$$\boxed{W_{\text{all}} = \Delta KE}$$

$$\boxed{W_{\text{ext}} + W_{\text{INC}} + W_{\text{IC}} = \Delta KE}$$

W_{ext} : Work done by external force

W_{INC} : Work done by internal non conservative force

W_{IC} : Work done by internal conservative force

Work done by internal conservative forces is also equal to negative of change in potential energy ($-\Delta PE$). Therefore

$$W_{\text{ext}} + W_{\text{INC}} - \Delta PE = \Delta KE$$

$$\boxed{W_{\text{ext}} + W_{\text{INC}} = \Delta KE + \Delta PE}$$

Work energy theorem (for constant force)

Consider a body of mass m having an initial velocity u. Under the action of a constant force (F) let the body attain a final velocity v as it undergoes a displacement S.

Using the equation of motion for a body under constant acceleration we get

$$v^2 - u^2 = 2aS$$

Multiplying both sides of the above equation with $m/2$ we get

$$\frac{m}{2}v^2 - \frac{m}{2}u^2 = \frac{m}{2} \times 2aS$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = maS$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = FS$$

$\frac{1}{2}mv^2$ is final kinetic energy

$\frac{1}{2}mu^2$ is initial kinetic energy

$F\ S$ is work done by the force

therefore

$$KE_f - KE_i = W$$

$$\boxed{W = \Delta KE}$$

Work energy theorem (for variable force)

Consider a body of mass m having an initial velocity u. Under the action of a force let the body attain a final velocity v.

Consider change in kinetic energy of a body as a function of time

$$\frac{\mathrm{d}KE}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}mv^2\right)$$

Assuming mass to be constant we get

$$\frac{\mathrm{d}KE}{\mathrm{d}t} = \frac{1}{2}m\frac{\mathrm{d}}{\mathrm{d}t}\left(v^2\right)$$

$$\frac{\mathrm{d}KE}{\mathrm{d}t} = \frac{1}{2}m2v\frac{\mathrm{d}v}{\mathrm{d}t}$$

$$\frac{\mathrm{d}KE}{\mathrm{d}t} = mv\frac{\mathrm{d}v}{\mathrm{d}t}$$

$$\frac{\mathrm{d}KE}{\mathrm{d}t} = ma\ v$$

$$\mathrm{d}KE = F\ \mathrm{d}x$$

To obtain total change in kinetic energy we integrate the above expression

$$\int_{\mathrm{i}}^{\mathrm{f}} \mathrm{d}KE = \int F\ \mathrm{d}x$$

$$KE_{\mathrm{f}} - KE_{\mathrm{i}} = W$$

$$\boxed{W = \Delta KE}$$